Structural Optimization of an Aircraft Wing Section Under a Constant Torque With MATLAB

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In the aerospace industry there is perpetual drive to manufacture lighter, cheaper, and stronger aircraft for both civilian and military application. Although this is heavily dependent on the current state of materials science, structural analysis and optimization are also powerful tools in advancing this goal. While performance is the top priority, financial aspects play a key role during the design process as well. Economic factors almost always correspond to design simplification and limiting the total volume of material needed for airframe components. Computer-aided engineering (CAE) software that implements finite element analysis (FEA) would usually be used to carry out the necessary calculations for optimization. However, Matlab's minimizing functions may also be used as a tool for carrying out design optimization and later visual CAE analysis can be carried out for verification.

1. INTRODUCTION

Structural optimization is a vital process that is almost always employed in the aerospace and mechanical engineering design process. In order to give a simplified demonstration of this process, a simple wing section can be examined. A diagram of the original wing section is shown in Appendix A, Fig. A.1. Two components, the shear flow, q, and the angle of twist per unit length, α, will be calculated and then optimized in order to see how efficient Matlab minimizes the values.

2. WING SECTION ANALYSIS

2.1 ASSUMPTIONS

During structural analysis of the wing section, some assumptions are made in order to simplify the analysis process:

1. The wing is composed of a single homogeneous material. This means we assume that material properties are constant (G = constant).
2. The displacement in each of the three cells will be equal. The angle of twist in each of the threes cells will be equal

2.2 DERIVATION OF EQUATIONS

Before any values can be optimized, the equations that govern this particular cross sections stress and displacement must be derived.

Torque: $$T = 2\sum_{i=1}^{n} q_i \bar{A}_i \qquad (1)$$

Angle of Twist: $$\theta = \frac{1}{2\bar{A}_i}\oint_{\Gamma_i}\frac{q}{Gt}ds \qquad (2)$$

Angle of Twist Per Unit Length: $$\alpha = \theta L \qquad (3)$$

From equations 1 and 2 above and the assumptions made, three equations will be derived with three unknowns which are the values for shear flow, q, in each cell.

These calculations are then done via matrix computations in Matlab which can be expressed:

$$\begin{bmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{bmatrix} * \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \frac{T}{2} \\ 0 \\ 0 \end{bmatrix} \qquad (4)$$

It is this basic computation that occurs in the main calculating M-file, "wingsec.m" that will be used along with an objective function, and an M-file containing our non-linear constraints. The non-linear constraints utilized for each part as follows:

1. The optimized shear flow, q in each cell must be less than the original optimized shear flow.
2. The optimized angle of twist per unit length, alpha, must be less than the original twist per unit length.

3 MATLAB OPTIMIZATION

3.1 ALGORITHM

After deriving the equations for finding the values for shear flow and twist per unit length by hand the following steps are taken in Matlab:

- Create a function that will calculate the shear flow and twist per unit length using the derived equations, and uses a data-array containing design parameters as input → wingsec.m
- Create a function that will serve as an object function for the *fmincon* function. It will output the values which we are trying to minimize (shear flow, q and twist per unit length alpha). → WSobj.m

- Define our non-linear constraints in an M-file that will also be implemented with the *fmincon* function. The non-linear constraints used will be: $q_{0i} \geq q_{opt}$, and $\alpha_0 \geq \alpha_{opt}$ or that the optimized values must be less than the original values. → WSnonlcon.m
- Employ the fmincon function which will minimize the defined values and output the new optimized values for the parameters. → WSfmincon.m
- Created a visual simulation with and without the optimized parameters for verification. → SolidWorks TM

4 RESULTS

An interesting result that can be seen below is the difference in the quality of optimization between the two studies. Because both values are dependent on one another one would assume that both would achieve similar results. This was not the case. By instructing Matlab to focus on the minimization of the shear flow rather than the twist per unit length (part 2) , a far better optimization was found.

4.1 STUDY 1: OPTIMIZING SYSTEM WITH RESPECT TO SHEAR FLOW

The first study conducted was to optimize the system for minimizing the shear flow. From Tbl. 1 it is apparent that the optimized values all found an optimum at exactly 30% of their original value. This is due to the bounds that were used with Matlab's *fmincon*. It was important to do this due to the linear relationship between thickness and shear flow. If it was left unbounded, certain thickness values would move towards zero while others would move towards infinity.

The values for shear flow and angle of twist per unit length both with and without optimum values are shown in Tbl. 2. It is exceptionally apparent that the optimization process was successful in minimizing not only the shear flow, but also the angle of twist (which we would assume). A simple static-linear simulation based on the optimized and optimized was carried out and is shown in Fig.1 and 2.

Table 1. Design parameters: original and optimized

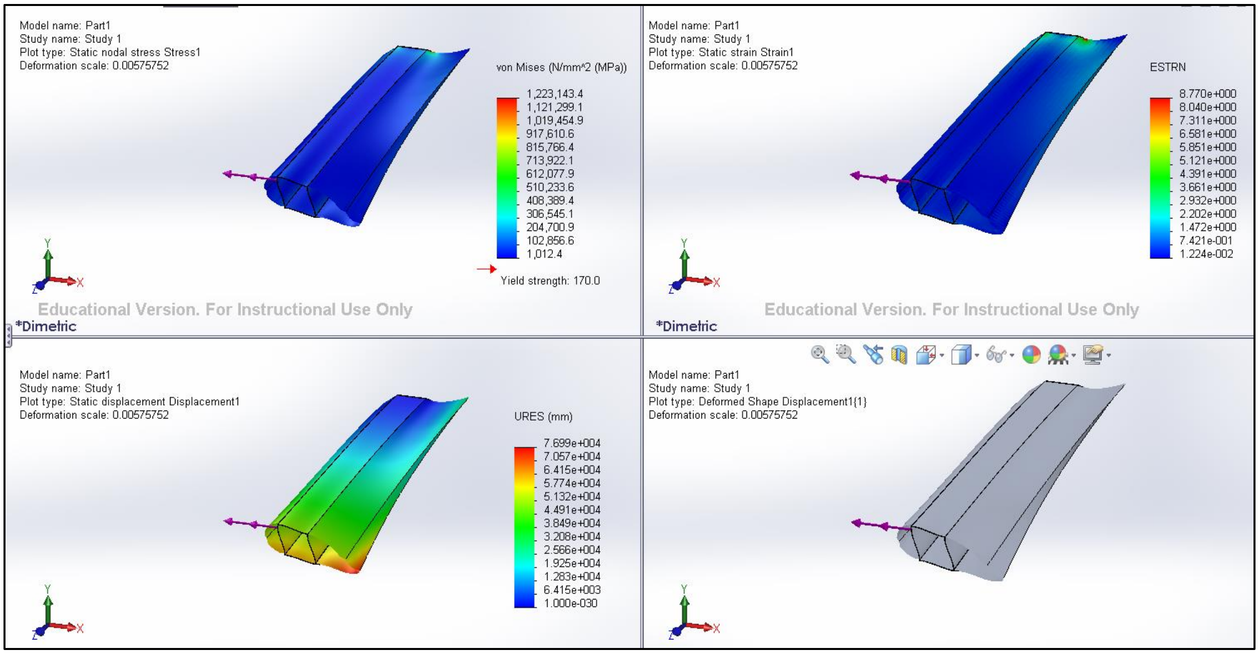
Parameter	Original Value	Optimized Value	Percent Change
t.BAC	2	1.4	0.3
t.BC	3	3.9	0.3
t.BD	4	5.2	0.3
t.CG	4	5.2	0.3
t.DG	3	2.1	0.3
t.DE	2	2.6	0.3
t.EF	2	2.6	0.3
t.FG	2	2.6	0.3
s.BAC	800	1040	0.3
s.BC	300	210	0.3
s.BD	400	280	0.3
s.CG	400	280	0.3
s.DG	300	390	0.3
s.DE	412	288.4	0.3
s.EF	100	70	0.3
s.FG	412	288.4	0.3
Area Nose	4.00E+04	28000	0.3
Area Center	1.20E+05	1.56E+05	0.3
Area Tail	8.00E+04	1.04E+05	0.3
Wingspan, L	4.00E+03	4000	0
Torque, T	1.00E+10	1.00E+10	0

Optimization Through Minimization of Shear Flow in Each Cell

Parameter	Before Optimization	After Optimization	Percent Change,%	Units
Shear flow, q_1	**12570**	**2549.3**	**+79.719**	N/mm
Shear flow, q_2	**25603**	**20467**	**+20.06**	N/mm
Shear flow, q_3	**17811**	**16690**	**+6.2939**	N/mm
Angle of twist, θ	**0.0018623**	**0.00066357**	**+64.368**	rad/mm
Angle of twist	**7.4493**	**2.6543**	**+64.368**	rad

Table 2. Shear flow and angle of twist per unit length computed values

Figure 1. Torsion simulation on **un-optimized** wing section



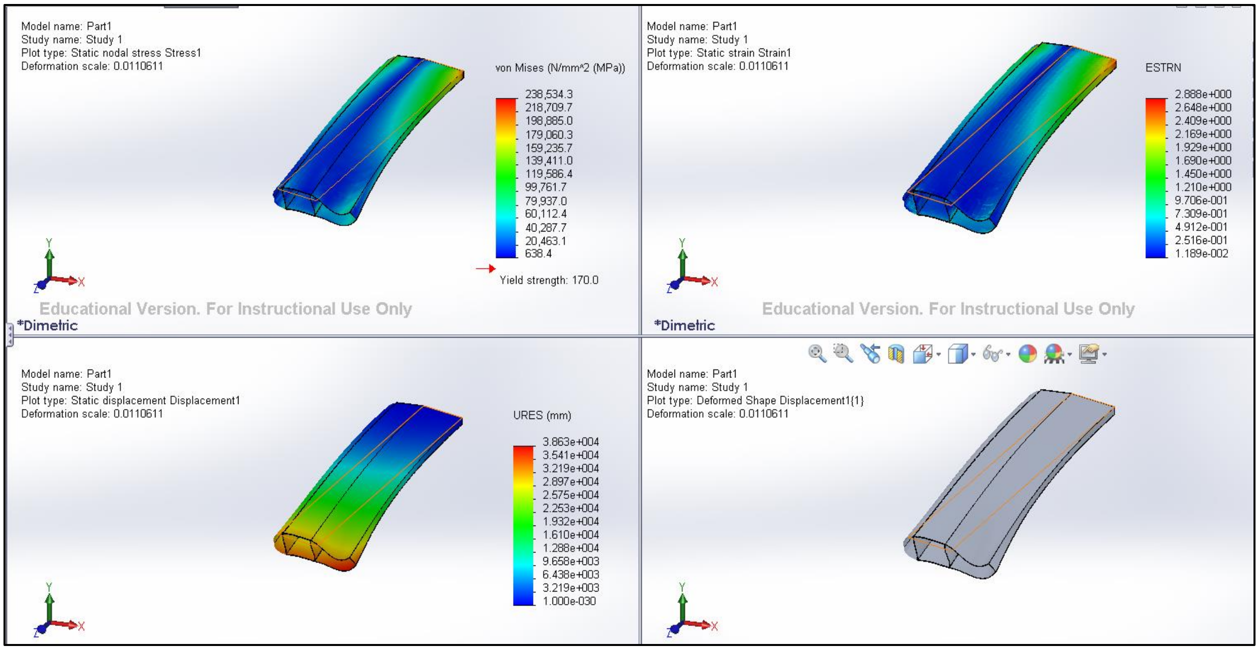
Model name: Part1
Study name: Study 1
Plot type: Static nodal stress Stress1
Deformation scale: 0.00575752

Model name: Part1
Study name: Study 1
Plot type: Static strain Strain1
Deformation scale: 0.00575752

Model name: Part1
Study name: Study 1
Plot type: Static displacement Displacement1
Deformation scale: 0.00575752

Model name: Part1
Study name: Study 1
Plot type: Deformed Shape Displacement1{1}
Deformation scale: 0.00575752

Figure 2. Torsion simulation on **optimized** wing section



Model name: Part1
Study name: Study 1
Plot type: Static nodal stress Stress1
Deformation scale: 0.0110611

Model name: Part1
Study name: Study 1
Plot type: Static strain Strain1
Deformation scale: 0.0110611

Model name: Part1
Study name: Study 1
Plot type: Static displacement Displacement1
Deformation scale: 0.0110611

Model name: Part1
Study name: Study 1
Plot type: Deformed Shape Displacement1{1}
Deformation scale: 0.0110611

4.2 STUDY 2: OPTIMIZING SYSTEM WITH RESPECT TO ANGLE OF TWIST PER UNIT LENGTH

The second portion of this study produced unexpected dissimilar results than in part one. It was expected that both optimizations would produce similar results and approximately the same values. This assumption did not hold true however as the original optimization produced a much more effective optimization for all members.

Optim

Table 3. Design parameters: original and optimized

it

Length

Parameter	Before Optimization	After Optimization	Percent Change. %	Units
Shear flow, q_1	**12570**	**16289**	**-29.586**	N/mm
Shear flow, q_2	**25603**	**24744**	**+3.3551**	N/mm
Shear flow, q_3	**17811**	**17213**	**+3.3575**	N/mm
Angle of twist, θ	**0.0018623**	**0.00096875**	**+47.981**	rad/mm
Angle of twist per unit length, α	**7.4493**	**2.7125**	**+63.587**	rad

Parameter	Original Value	Optimized Value	Percent Change
t.BAC	2	2.6	0.3
t.BC	3	2.1	0.3
t.BD	4	5.2	0.3
t.CG	4	5.2	0.3
t.DG	3	3.9	0.3
t.DE	2	2.6	0.3
t.EF	2	2.6	0.3
t.FG	2	2.6	0.3
s.BAC	800	716.94	0.103825
s.BC	300	390	0.3
s.BD	400	280	0.3
s.CG	400	280	0.3
s.DG	300	243.06	0.1898
s.DE	412	288.4	0.3
s.EF	100	70	0.3
s.FG	412	288.4	0.3
Area Nose	4.00E+04	40003	0.000075
Area Center	1.20E+05	1.21E+05	0.008341667
Area Tail	8.00E+04	8.00E+04	0.0000875
Wingspan, L	4.00E+03	3768.9	0.057775
Torque, T	1.00E+10	1.00E+10	0

Table 4. Shear flow and angle of twist per unit length

Table 5. Shear stress in each section of the wing

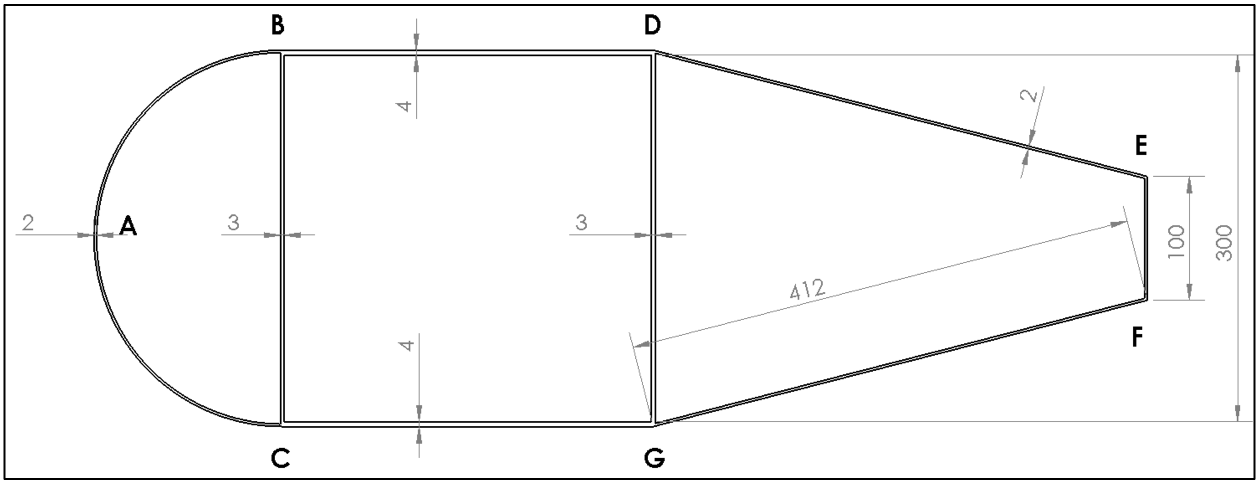
Shear Stress, Tau			
	Minimize Shear Flow		
Section	Before Optimization	After Optimization	Percent Change, %
BAC	6285	1821	71.02625298
BC	-4344.3	-4594.3	5.754667035
BD	6400.7	3936	38.50672583
CG	6400.7	3936	38.50672583
DG	2597.4	1798.5	30.75768076
DE	8905.4	6419.3	27.9167696
EF	8905.4	6419.3	27.9167696
FG	8905.4	6419.3	27.9167696
	Minimize Angle of Twist Per Unit Length		
Section	Before Optimization	After Optimization	Percent Change, %
BAC	6285	6265.1	0.316626889
BC	-4344.3	-4026	7.326842069
BD	6400.7	4758.4	25.65813114
CG	6400.7	4758.4	25.65813114
DG	2597.4	1930.9	25.66027566
DE	8905.4	6620.5	25.65746626
EF	8905.4	6620.5	25.65746626
FG	8905.4	6620.5	25.65746626

5 CONCLUSIONS

Although structural optimization with CAE analysis may be a faster method, Matlab is also capable of completing the task with a little more manual computation involved. An interesting and unexpected result of this study is the difference in quality of optimization between the two studies. One would typically assume that if two properties are dependent on one another than it would not matter which one is optimized. This notion was disproven in this study as part one produced a superior system optimization.

APPENDIX A

Figure A.1 – Original wing cross section with dimensions in mm



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